

Removing Ambiguities in the Neutrino Mass Matrix

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We suggest that the weak-basis independent condition $\det(M_\nu) = 0$ for the effective neutrino mass matrix can be used in order to remove the ambiguities in the reconstruction of the neutrino mass matrix from input data available from present and future feasible experiments. In this framework, we study the full reconstruction of M_ν with special emphasis on the correlation between the Majorana CP-violating phase and the various mixing angles. The impact of the recent KamLAND results on the effective neutrino mass parameter is also briefly discussed.

1 Introduction

Neutrino masses and mixings are most likely described by a Majorana mass matrix which naturally arises in the context of the standard electroweak gauge theory, with the implicit assumption that $B - L$ is violated at a high-energy scale. The smallness of neutrino masses is then elegantly explained by the seesaw mechanism [1]. The 3×3 complex symmetric Majorana neutrino mass matrix M_ν contains nine physical parameters, while realistic experiments can determine only seven independent quantities. This leads to the wretched situation that no set of feasible experiments can fully determine the neutrino mass matrix. This basic observation has encouraged Frampton, Glashow and Marfatia [2] to propose that the neutrino mass matrix M_ν contains texture zeros, in order to reduce the number of free parameters. However, the presence of zeros in M_ν crucially depends on the weak basis one chooses. Therefore, it is desirable to consider basis-independent constraints on the neutrino mass matrix.

In this letter we address the question of whether it is possible to achieve an appropriate reduction of parameters through the introduction of a weak-basis independent condition. We propose the basis independent condition that the

determinant of the neutrino mass matrix vanishes, that is $\det(M_\nu) = 0$. Since this condition gives two constraints on the parameters, the neutrino mass matrix has now just 7 parameters which can be fully determined by future feasible experiments. We note that the fact that $\det(M_\nu) = 0$ implies the elimination of two parameters has to do with the assumed Majorana nature of neutrinos. In contrast, if one imposes the condition $\det(M_u) = 0$ in the quark sector, one loses only one parameter in the electroweak sector, since the CKM matrix continues having four parameters even in the limit $m_u = 0$. On the other hand, the condition $m_u = 0$ allows to remove another parameter from the theory, since it implies $\bar{\theta} = 0$ ($\bar{\theta}$ is the coefficient of $F_{\mu\nu}\tilde{F}_{\mu\nu}$), thus providing a possible solution to the strong CP problem [3]. It is also interesting that the Affleck-Dine scenario for leptogenesis [4] requires the mass of the lightest neutrino to be $m_1 \simeq 10^{-10}$ eV [5,6], which leads practically to our condition $\det(M_\nu) = 0$. Furthermore, such an extremely small mass may be explained by a discrete Z_6 family symmetry [6]. Within the framework of the seesaw mechanism, the study of leptonic CP violation at high energies and its relation to the neutrino mass spectrum could have profound cosmological implications, for instance in the generation of the observed baryon asymmetry of the universe through leptogenesis [7,8]. However, in this letter we will restrict ourselves to low energies and therefore our analysis remains valid independently of the high energy origin of the effective neutrino mass matrix.

In the framework where $\det(M_\nu) = 0$, we study the full reconstruction of M_ν with special emphasis on the correlation between the Majorana CP-violating phase and the various mixing angles. We also discuss how future neutrino-less double beta decay experiments could invalidate the assumed weak-basis independent condition.

2 Reconstruction of the neutrino mass matrix

Let us start by summarizing the present data on neutrino mass-squared differences and mixing angles, obtained from the evidence for neutrino oscillations in atmospheric, solar and reactor neutrino experiments. Assuming two-neutrino mixing and dominant $\nu_\mu \rightarrow \nu_\tau$ ($\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$) oscillations, the atmospheric neutrino data obtained from Super-Kamiokande experiments yields at 99.73 % C.L. [9]:

$$\begin{aligned} 1.5 \times 10^{-3} \text{ eV}^2 &\leq \Delta m_{\text{at}}^2 \leq 5.0 \times 10^{-3} \text{ eV}^2, \\ \sin^2 2\theta_{\text{at}} &> 0.85, \end{aligned} \tag{1}$$

with the best-fit values $(\Delta m_{\text{at}}^2)_{BF} = 2.5 \times 10^{-3} \text{ eV}^2$ and $(\sin^2 2\theta_{\text{at}})_{BF} = 1$. On the other hand, global neutrino analyses of the solar neutrino data [10] under the assumption of $\nu_e \rightarrow \nu_{\mu,\tau}$ oscillations/transitions favors the LMA

$$\begin{aligned} 2.2 \times 10^{-5} \text{ eV}^2 &\leq \Delta m_{\odot}^2 \leq 2.0 \times 10^{-4} \text{ eV}^2, \\ 0.18 &\lesssim \sin^2 \theta_{\odot} \lesssim 0.37, \end{aligned} \quad (2)$$

and the best-fit values $(\Delta m_{\odot}^2)_{BF} = 5 \times 10^{-5} \text{ eV}^2$ and $(\sin^2 \theta_{\odot})_{BF} = 0.25$. Finally, the reactor neutrino data obtained from the CHOOZ experiment [11] puts an upper bound on the leptonic mixing matrix element U_{e3} . The combined three-neutrino oscillation analyses of the solar, atmospheric and reactor data imply at 99.73 % C.L. [12,13]:

$$|U_{e3}| < 0.22, \quad (3)$$

with a best-fit value of $(|U_{e3}|)_{BF} \simeq 0.07$ found in [12].

Another important input information comes from neutrinoless double β decay experiments, which could provide us with evidence for non-vanishing CP-violating Majorana phases, and thus, for the Majorana nature of massive neutrinos [14]. For Majorana neutrino with masses not exceeding a few MeV, the amplitude of this process is proportional to the so-called effective Majorana mass parameter $m_{ee} = |(M_{\nu})_{11}|$. Although no evidence for $(\beta\beta)_{0\nu}$ decay has been found so far, rather stringent upper bounds have been obtained. In particular, the ^{76}Ge Heidelberg-Moscow experiment has reported the limit $m_{ee} < 0.35 \text{ eV}$ at 90 % C.L. and the IGEX collaboration, $m_{ee} < (0.33 - 1.35) \text{ eV}$ at 90 % C.L. . A considerable higher sensitivity is expected in future experiments. For instance, values of $m_{ee} \simeq 5.2 \times 10^{-2}$ (EXO), $m_{ee} \simeq 3.6 \times 10^{-2}$ (MOON) and $m_{ee} \simeq 2.7 \times 10^{-2}$ (CUORE) are planned to be achieved [15].

As far as the CP-violating Dirac phase is concerned, there is at present no experimental information on its value. However, neutrino factories will in principle be able to measure $\text{Im}(U_{e2}U_{\mu 3}U_{\mu 2}^*U_{e3}^*)$ and thus determine the Dirac phase in any specific parametrization of the leptonic mixing matrix U .

In our framework where $\det(M_{\nu}) = 0$, the neutrino mass matrix is characterized by seven parameters and therefore one can use the above described seven inputs from experiment to fully reconstruct M_{ν} in the weak basis where the charged lepton mass matrix is diagonal and real. In this basis, M_{ν} can be written as:

$$M_{\nu} = U^* \text{diag}(m_1, m_2 e^{i\alpha_2}, m_3 e^{i\alpha_3}) U^{\dagger}, \quad (4)$$

where m_i are the moduli of the light neutrino masses, α_i are the two Majorana phases and U is the MNS neutrino mixing matrix, which we choose to

parametrize in the form:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & -c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13}e^{i\delta} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}e^{i\delta} \end{pmatrix}, \quad (5)$$

where $c_{ij} \equiv \cos \theta_{ij}$, $s_{ij} \equiv \sin \theta_{ij}$, $0 \leq \theta_{ij} \leq \pi/2$ and δ is the CP-violating Dirac phase. The above parametrization turns out to be more convenient in the analysis of the effective mass parameter m_{ee} , since in this case this matrix element depends only on the Majorana phases α_i and not on the Dirac phase δ . If one uses instead the standard parametrization [16], $U_\nu = U \text{diag}(1, 1, e^{-i\delta})$, then the phase δ would enter in the combination $\alpha_3 - 2\delta$.

The condition $\det(M_\nu) = 0$ together with the experimental constraints on Δm_{\odot}^2 and Δm_{\oplus}^2 imply that only one neutrino can have a vanishing mass. By identifying the indexes 12 and 23 with the solar and atmospheric neutrinos, respectively, two possible scenarios can be distinguished. In the first case, the diagonal matrix in Eq. (4) is of the form

$$\text{diag}(0, m_2 e^{i\alpha}, m_3) \quad (\text{Case I}), \quad (6)$$

while in the second one, the above matrix is

$$\text{diag}(m_1, m_2 e^{i\alpha}, 0) \quad (\text{Case II}), \quad (7)$$

with the relevant Majorana phase given in both cases by $\alpha = \alpha_2 - \alpha_3$.

Since it is for the matrix element m_{ee} that we have direct experimental access, from now on we will restrict our analysis to the implications of our assumptions in the determination of this parameter and, consequently, of the Majorana phase α [17].

2.1 Case I: Standard hierarchy

In this case $m_2 = \sqrt{\Delta m_{\odot}^2}$ and $m_3 = \sqrt{\Delta m_{\oplus}^2 + \Delta m_{\odot}^2}$ and it follows from Eq. (4) that

$$m_{ee}^2 \equiv |(M_\nu)_{11}|^2 = m_2^2 |U_{12}|^4 + m_3^2 |U_{13}|^4 + 2 m_2 m_3 |U_{12}|^2 |U_{13}|^2 \cos \alpha. \quad (8)$$

In the parametrization (5) we have then

$$m_{ee}^2 = m_2^2 c_{13}^4 s_{12}^4 + m_3^2 s_{13}^4 + 2 m_2 m_3 c_{13}^2 s_{13}^2 s_{12}^2 \cos \alpha, \quad (9)$$

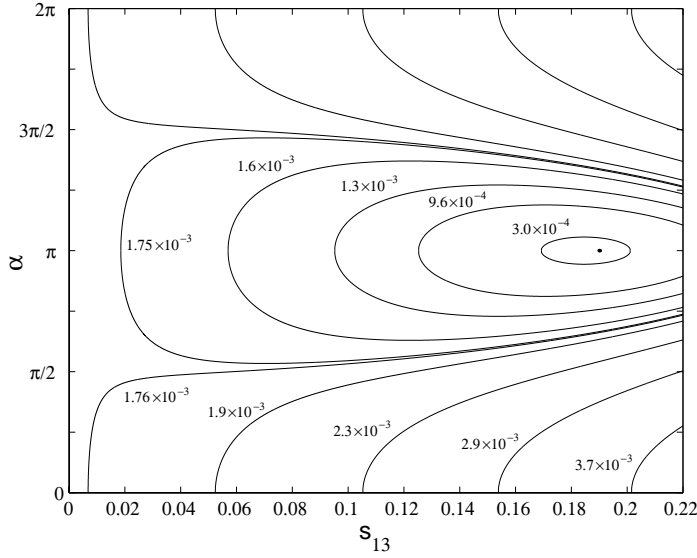


Fig. 1. Contour plots of the Majorana phase α as a function of s_{13} for different values of the neutrinoless double β -decay parameter m_{ee} . The black dot corresponds to $m_{ee} = 0$. The best-fit values of s_{12} , Δm_{\odot}^2 and Δm_{\oplus}^2 have been used.

with the upper and lower bounds given by

$$m_{ee}^{\text{upper}} = |m_2 s_{12}^2 + s_{13}^2 (m_3 - m_2 s_{12}^2)|, \quad (10)$$

$$m_{ee}^{\text{lower}} = |m_2 s_{12}^2 - s_{13}^2 (m_3 + m_2 s_{12}^2)|, \quad (11)$$

which correspond to $\alpha = 0$ (same CP parity) and $\alpha = \pi$ (opposite CP parities), respectively. We notice from Eq. (11) that cancellations in m_{ee} can occur if

$$\tan^2 \theta_{13} = \frac{m_2 s_{12}^2}{m_3} \simeq \frac{\Delta m_{\odot}^2 s_{\odot}^2}{\Delta m_{\oplus}^2}. \quad (12)$$

Using the experimental ranges of Eqs. (1)-(3), we can get the limits on s_{13} for such cancellations to occur. We find $0.12 \lesssim U_{e3} \lesssim 0.22$.

The Majorana phase α can be extracted from Eq. (9), leading to:

$$\cos \alpha = \frac{m_{ee}^2 - m_3^2 s_{13}^4 - m_2^2 c_{13}^4 s_{12}^4}{2 m_2 m_3 c_{13}^2 s_{13}^2 s_{12}^2}. \quad (13)$$

In Fig. 1 we present the contour plots of α as a function of s_{13} for different values of the neutrinoless double β -decay parameter m_{ee} . We consider the best-fit values of Δm_{\oplus}^2 and s_{12} , Δm_{\odot}^2 for the LMA MSW solar solution (see Eqs. (1) and (2)).

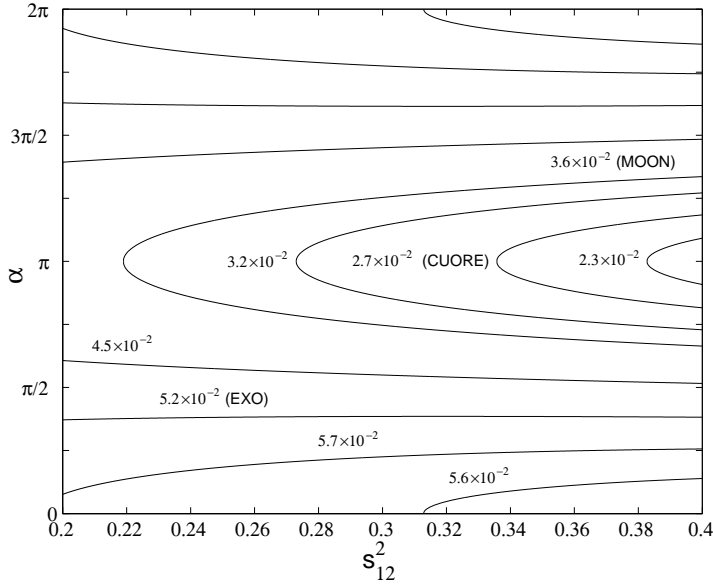


Fig. 2. Contour plots of the Majorana phase α as a function of s_{12}^2 for different values of the neutrinoless double β -decay parameter m_{ee} . The contours corresponding to the sensitivity of the future MOON, CUORE and EXO $(\beta\beta)_{0\nu}$ -decay experiments have also been included. The best-fit values of s_{13} , Δm_{\odot}^2 and Δm_{\oplus}^2 have been used.

2.2 Case II: Inverted hierarchy

In this case $m_1 = \sqrt{\Delta m_{\oplus}^2}$ and $m_2 = \sqrt{\Delta m_{\oplus}^2 + \Delta m_{\odot}^2}$. For the effective mass parameter m_{ee} we obtain from Eq. (4)

$$m_{ee}^2 \equiv |(M_{\nu})_{11}|^2 = m_1^2 |U_{11}|^4 + m_2^2 |U_{12}|^4 + 2 m_1 m_2 |U_{11}|^2 |U_{12}|^2 \cos \alpha, \quad (14)$$

which in the parametrization (5) is equivalent to

$$m_{ee}^2 = c_{13}^4 (m_1^2 c_{12}^4 + m_2^2 s_{12}^4 + 2 m_1 m_2 s_{12}^2 c_{12}^2 \cos \alpha). \quad (15)$$

The upper and lower bounds of m_{ee} are given by

$$m_{ee}^{\text{upper}} = c_{13}^2 (m_1 c_{12}^2 + m_2 s_{12}^2), \quad \text{for } \alpha = 0, \quad (16)$$

$$m_{ee}^{\text{lower}} = c_{13}^2 |m_1 c_{12}^2 - m_2 s_{12}^2|, \quad \text{for } \alpha = \pi. \quad (17)$$

The vanishing of m_{ee} requires $\tan^2 \theta_{12} = m_1/m_2 \simeq 1$. Since values of $\tan^2 \theta_{12} \simeq 1$ are excluded by the present LMA solar data, such cancellations cannot occur in this case.

From Eq. (15) we find:

$$\cos \alpha = \frac{m_{ee}^2 - m_1^2 c_{12}^4 c_{13}^4 - m_2^2 s_{12}^4 c_{13}^4}{2 m_1 m_2 s_{12}^2 c_{12}^2 c_{13}^4}. \quad (18)$$

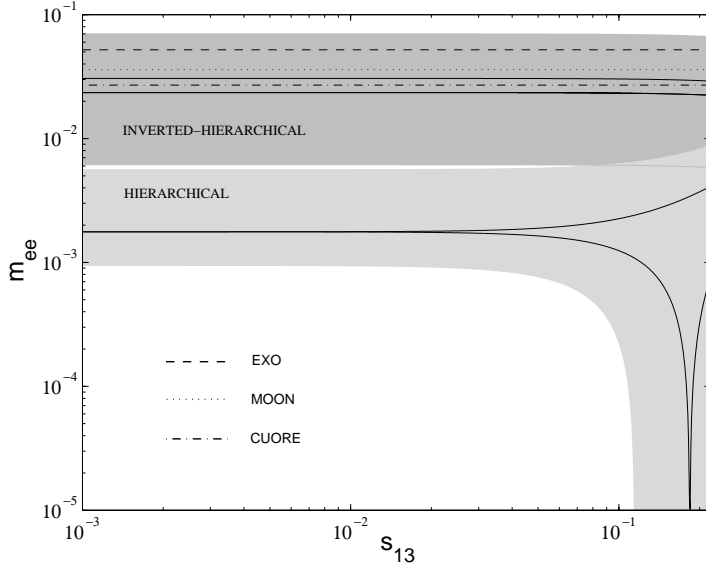


Fig. 3. Allowed regions for the effective mass parameter m_{ee} in the cases of hierarchical (light grey) and inverted-hierarchical (dark grey) neutrino mass spectra, assuming $\det(M_\nu) = 0$ and taking into account all the available experimental data for solar, atmospheric and reactor neutrinos as given in (1)-(3). The areas delimited by the solid lines correspond to the allowed regions considering the best-fit values of Δm_\odot^2 , Δm_\oplus^2 and θ_\odot .

We notice that the Majorana phase α is not very sensitive to the small values of $U_{e3} = s_{13}$ allowed by the present data. On the other hand, the value of α is more sensitive to the solar mixing angle θ_{12} . In Fig. 2 we present the contour plots of the Majorana phase α as a function of s_{12}^2 for different values of the neutrinoless double β -decay parameter m_{ee} . The values of m_{ee} are in this case at the reach of the future $(\beta\beta)_{0\nu}$ -decay experiments. For comparison, the contours corresponding to the sensitivity of the future MOON, CUORE and EXO experiments are also plotted. We have used the best-fit values of s_{13} , Δm_\odot^2 and Δm_\oplus^2 .

In Fig. 3 we present the allowed regions for the effective mass parameter m_{ee} in the cases of hierarchical (Case I) and inverted-hierarchical (Case II) neutrino mass spectra, assuming the weak-basis independent condition $\det(M_\nu) = 0$. We use the experimental ranges for solar, atmospheric and reactor neutrinos as given in Eqs. (1)-(3). The areas delimited by the solid lines correspond to the allowed regions if one uses the best-fit values of Δm_\odot^2 , Δm_\oplus^2 and θ_\odot . This figure also illustrates how future $(\beta\beta)_{0\nu}$ -decay experiments could in principle distinguish the two cases considered, or even exclude one or both of them. In particular, it is seen from the figure that if $m_{ee} \lesssim 10^{-3}$ and $U_{e3} \lesssim 0.05$ then the vanishing of the determinant of M_ν is not a viable assumption. It is also clear that a better knowledge on the mixing angles and Δm^2 's is crucial to distinguish between the hierarchical and inverted-hierarchical cases.

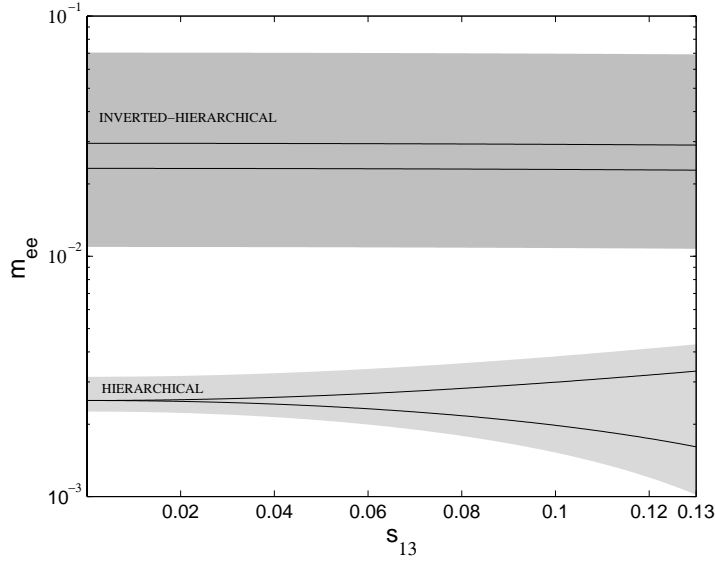


Fig. 4. Allowed regions for the effective mass parameter m_{ee} in the cases of hierarchical (light grey) and inverted-hierarchical (dark grey) neutrino mass spectra, assuming $\det(M_\nu) = 0$ and taking into account the $\pm 1\sigma$ estimates for Δm_{12}^2 , s_{12}^2 and s_{13} obtained from the global analysis of the pre-KamLAND and new KamLAND data given in [20]. The areas delimited by the solid lines correspond to the allowed regions considering the best-fit values $\Delta m_{12}^2 = 7 \times 10^{-5} \text{ eV}^2$ and $s_{12}^2 = 0.3$.

To conclude our analysis let us briefly comment on the recent results reported by the Kamioka Liquid Scintillator Antineutrino Detector (KamLAND) [19]. These results constitute the first terrestrial evidence of the solar neutrino anomaly. The observation of $\bar{\nu}_e$ disappearance reinforces the interpretation of the previous neutrino data through $\nu_e \rightarrow \nu_{\mu,\tau}$ oscillations. The combined analysis of the KamLAND spectrum with the existent solar and terrestrial data already exclude some portions of the allowed region in the $(\Delta m_{12}^2, s_{12}^2)$ -plane for the LMA solution [20–22]. One of the consequences of these global analyses is the splitting of the LMA region in two sub-regions. In Fig. 4 we plot the allowed regions for the effective neutrino mass parameter m_{ee} under the initial assumption $\det(M_\nu) = 0$ and taking as an example the $\pm 1\sigma$ estimates $\Delta m_{12}^2 \simeq (7.3 \pm 0.8) \times 10^{-5} \text{ eV}^2$, $s_{12}^2 \simeq 0.315 \pm 0.035$ and $s_{13} \lesssim 0.13$ given in Ref. [20]. In this case there is no overlap between the two regions corresponding to the hierarchical and inverse-hierarchical spectra of the light neutrinos. Moreover, it is noticeable that cancellations on m_{ee} are no longer present. This example also reveals the importance of future neutrino data in removing the ambiguities of the neutrino mass matrix.

3 Conclusions

In order to remove the ambiguities in the reconstruction of the neutrino mass matrix from input data, we have proposed in this letter to use the weak-basis independent condition that the determinant of the effective neutrino mass matrix vanishes. Since the condition $\det(M_\nu) = 0$ gives two additional constraints on the parameters, the resulting mass matrix has only 7 independent quantities which can be fully determined by future feasible neutrino experiments. We have focused our analysis on the correlation between the Majorana CP-violating phase and the various mixing angles. In particular, we have discussed how future neutrinoless double beta decay experiments could invalidate the above weak-basis independent condition. Finally, we have also discussed the impact of the recent KamLAND results on the effective neutrino mass parameter.

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